Spinodal instabilities within BUU approach

S. Chattopadhyay

S. N. Bose Centre for Basic Sciences, DB-17, Bidhannagore, Calcutta-700064, India

Abstract

Using a recently developed method for the inclusion of fluctuation in the BUU dynamics, we study the self-consistent propagation of inherent thermal noise of unstable nuclear matter. The large time behaviour of the evolving system exhibits synergism between fluctuation and non-linearities in a universal manner which manifest in the appearance of macroscopic structure in the average description.

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In recent years a considerable interest has grown in the understanding of the role of fluctuation in the dynamics of multifragmentation. In the decompression phase of the collision, the density of the composite nuclear system eventually attains such a low value so that it crosses the boundary of the so called spinodal zone where a small perturbation can grow rapidly until the fragments form. The fluctuation certainly plays a significant role in the process of fragmentation. To unravel some basic features of such delicate process simple model study seems encouraging. For this purpose, different theoretical approaches based on Landau [1,2] or Boltzmann-Uhling-Ulhenbeck (BUU) [3] equation have been utilized.

Let us concentrate on the BUU approach to the problem. In this model, the description of the process is governed by the evolution of reduced one body density distribution. In the mean field approximation, the fluctuations arising from the stochastic part of the collision term are ignored and evolution of a single average effective density is considered only. In this description the bundle of trajectories are confined in a certain region of the associated phase space. The presence of collision term in the equation corresponds to the dissipative effect in the evolution. When the system reaches the point of instability where any fluctuation, however, small it is, amplifies indefinitely so that trajectories may bifurcate and drive the system to some new fixed point. In other words, different manifestation of the system are now become accessible dynamically. Therefore, the mean trajectory description has been lost. To encounter such a situation, the general strategy is to study the evolution of a collection of initially prepared samples. Due to the very presence of the stochasticity in the process their fates are different. To make this picture consistent, a usual method is to introduce a fluctuation term in the equation of motion, so that the nature of the solution become probabilistic. Within the framework of BUU equation this prescription is properly incorporated and the resultant evolution equation for phase space density is commonly known as the Boltzmann-Langevin (BL) equation [4].

Guided by the same physical principle, several computational schemes have been proposed to incorporate fluctuation in the BUU equation [5,6]. In this context, we want to mention a very recent development (see Ref. [7] where a novel method of simulation is pro-

posed to incorporate fluctuations in the BUU equation. The occupancy factor $n(\mathbf{r},\mathbf{p})$ at a particular elementary cell $\Delta\mathbf{s}$ of size $\Delta\mathbf{r}\Delta\mathbf{p}/h^D$ (D the dimension of the physical space) around a point $(\mathbf{r},\mathbf{p})$ in the associated phase space of the one body description of the system is considered to be discrete in nature so that it takes the value 1 (say success) or 0 (failure) otherwise. The basic transition probability (say, $12 \rightarrow 1'2'$) within a small interval of time is given by $\mathcal{T}(12 \rightarrow 1'2') = \omega_{12\rightarrow 1'2'}\Delta t\Delta\mathbf{s}^4 n_1 n_2 \bar{n}_{1'}\bar{n}_{2'}$ where $\omega_{12\rightarrow 1'2'}\Delta\mathbf{s}^4$ is the basic transition rate. For a particular transition, every sample is tried with a probability of success $\omega\Delta t\Delta\mathbf{s}^4$. However due to the presence of four occupancy factors in the above relation a successful transition occur when an appropriate configuration of particle and hole states appeared in a sample. By this way enough fluctuation in sample space can be generated and at the same time, the relevant statistical criteria i.e. $\sigma_i^2 = \langle n\rangle_i\langle\bar{n}\rangle_i$ can be fulfilled exactly for every cell i.

Another essential aspect of the simulation is to generate the solution of the Vlasov part of the BUU equation in the presence of the mean field potential. Due to the discrete nature of the phase space occupancy $n(\mathbf{r},\mathbf{p})$, the usual procedure of matrix method [3,5] cannot be applied here properly. To consider Vlasov propagation of phase space distribution we follow the Nordheim's approach, closely related to the well known test particle method of solving Vlasov equation. A filled unit phase cell may be considered as a particle. The time evolution of these particles can be described by the standard leap-frog algorithm in the following manner,

$$X_k^i(t+\Delta t) = X_k^i(t) + \frac{1}{m}P_k^i\Delta t \ ; \tag{1}$$

$$P_k^i(t+\Delta t) = P_k^i(t) + \nabla_k\mathcal{U}(\rho(\mathbf{r},t))\Delta t \tag{2}$$

where $(X_k^i(t), P_k^i(t))$ represent the kth component of position and momentum of the ith particle at time t. It is important to note that unlike to the test particle method the position and also momentum of the particle are represented by the grid points here, therefore, its evolution with time take places in discrete steps. However to provide a reasonably good energy conserving evolution, we incorporate a modification in the leap-frog routine. The

error comes mainly due to the rounding off the quantities X_k^i/x_d or P_k^i/p_d where x_d and p_d are the co-ordinate and momentum grid spacing. The accumulated error that arise in momentum due to the incoming particles at a particular spatial cell (say l) is given as $\Delta\tilde{p}_k^l = \sum_i \delta p_k^i$. Here δp_k^i is the residual part of P_k^i so that $P_k^i = p_d \times$ nearest integer of$(P_k^i/p_d) + \delta\tilde{p}_k^i$. Therefore, to reduce this error substantially, one may assume that the momentum of all the particles in a cell l are modified as $p_d \times$ nearest integer of$(\frac{P_k^i}{p_d}) + \frac{\Delta\tilde{p}_k^l}{N_l}$. Here N_l is the total number of incoming particles in cell l. To check the reliability of this method we consider the evolution of the breathing mode of a spherical nucleus in 2-dimension having small skin of about 2 fm. We observe two interesting points: 1) the initial spherical symmetry in momentum and co-ordinate space are maintained exactly and 2) no two particles appear simultaneously in the same phase cell throughout the evolution. However, it is to be noted that when fluctuation in phase cell is taken into account by allowing the the collisions among the particles the above mentioned features are gradually lost. In this situation, the smearing of the distribution becomes necessary. This is incorporated by simply moving a particle in the neighbouring sites in momentum space if it tries to access a cell in the process of evolution which is already filled. In this simulation, the values of x_d and p_d are taken as 1 fm. and 40$\sim$60 MeV/c respectively. The total energy per particle is found to be conserved up to 95% even at time $t = 100$ fm/c.

Using the above mentioned simulation procedure the present study revisits the problem of nuclear matter in spinodal zone. This work mainly concerned about the large time behaviour of the system and the non-linearities that appear in the process of evolution. To make this study consistent with earlier investigation we adopt the scenario that was studied rigorously by several authors [3,5] namely, a gas of fermions situated inside a two dimensional torus. For the effective one body field at two dimensional grid point (x,y) we employ a simplified Skyrme interaction $U(x,y) = A\frac{\rho(x,y)}{\rho_0} + B(\frac{\rho(x,y)}{\rho_0})^2$ with $A = -100.3$ MeV, $B = 48$ MeV, the saturation density $\rho_0 = 0.55$ fm^{-2}. To evaluate effective potential on the lattice an averaging over 2-dimensional Gaussian function having width 0.87 is done. The in-medium cross section is taken as 2.4 fm and Fermi-momentum $P_f = 260$ MeV/c. For simulation

we consider the square lattice of size (21×21) in co-ordinate space having width $x_d = 1$ fm., (31×31) in momentum space with $p_d = 40$ MeV/c and the number of samples $\mathcal{N}_s$ for parallel runs are taken to be 100. To ensure that our system is initially situated inside the spinodal zone we prepare the samples having uniform (in space) average density $\langle\rho\rangle$ (or the number density $\langle N\rangle$)=$0.5\rho_0$ (N_0) and momentum distribution same as that of a fermi gas at temperature $T = 3$ MeV.

At a given temperature $T \neq 0$ there exists thermal fluctuation in the initial state. To incorporate this important feature within our simulation procedure we prepare the samples with the assumption that the distribution of N is given by a Gaussian function with mean $\langle N\rangle$ where as the sample average value of occupancy $\langle n\rangle_i$ is given by the fermi distribution of appropriate temperature T. The variance σ_N^2 satisfies the standard relation $\sigma_N^2 = \sum_N \sum \sigma_i^2 = \sum_N \sum \langle n\rangle_i \langle \bar{n}\rangle_i$. To maintain spatial uniformity in initial state, the value of N or the momentum distribution is considered to be the same at every spatial cell of a particular sample. Inspite of the fact that at $T = 0$ all collisions are Pauli blocked, the symmetry of the system remains intact even by BL treatment used here or the same used in Ref. [3]. It is to be noted that in this case $\sigma_N^2 = 0$ i.e. all samples are identical. The presence of the thermal fluctuation in the initial state is in fact, the source of disturbance which generates irregularities subsequently in co-ordinate space and may amplify due to the action of the mean field. Hence, to make the picture consistent, the preparation of the samples in the aforesaid manner is therefore an essential part of our simulation procedure.

Let us now turn to the investigation of the Boltzmann-Langevin dynamics on lattice. The normal mode analysis of the evolving density will allow us to study the interplay between instabilities and fluctuations. Following the Ref. [3] we introduce two point correlation function to the associated density fluctuation as

$$\sigma_{\mathbf{k}}(t) = \langle|\delta\rho_{\mathbf{k}}(t)|^2\rangle = \frac{1}{L^4}\int^{L^2} d\mathbf{r}' \int^{L^2} d\mathbf{r} e^{-\mathbf{k}.|\mathbf{r}'-\mathbf{r}|}\langle\delta\rho(\mathbf{r}')\delta\rho(\mathbf{r})\rangle \tag{3}$$

For theoretical analysis of the time evolution of the quantity σ_k the starting point is to linearise the Vlasov equation and derive the equation for normal modes (for example see Ref.

[8]). In the presence of fluctuation in the transport process the evolution of the amplitude of the Fourier components (or in other words, that of the collective mode ν) can be represented by the following Langevin equation

$$\frac{dA_\nu(t)}{dt} = i\omega_\nu A_\nu(t) + \tilde{\mathcal{B}}_\nu(t) \tag{4}$$

where A_ν is the two component amplitude matrix (with elements A_ν^+ and A_ν^-) of the Fourier components of the density fluctuation δf_ν so that $\delta f_\nu = A_\nu^+ e^{i\omega_\nu t} + A_\nu^- e^{-i\omega_\nu t}$. $\tilde{\mathcal{B}}_\nu(t)$ is the kicking term of the Langevin equation. The associated correlation or diffusion matrix for Gaussian Markov process can be given by a Hermition matrix with components

$$\langle \mathcal{B}_\nu^i(t)\mathcal{B}_\nu^j(t')\rangle = \mathcal{D}_\nu^{ij}\delta(t-t') \tag{5}$$

where i (j) runs for states '+' or '-'.

Let us now concentrate on the situation of instability. For infinite system one can find a range of ν , for which all the modes are unstable, the associated the frequencies $\omega_\nu(= -i\Omega_\nu)$ are purely an imaginary numbers and beyond that range all modes are stable. Therefore, in course of time the fate of the system is dictated by the evolution of few such most unstable modes. The probabilistic evolution of A_μ (see eq.(4)) can be described as well in terms of Fokker-Planck equation. The evolution equations for first and second moment related to it can be written as [9]

$$\frac{dA_\mu}{dt} = \begin{pmatrix} \Omega_\nu & 0 \\ 0 & -\Omega_\nu \end{pmatrix} \langle A_\mu\rangle(t) \tag{6}$$

$$\frac{d\sigma_\nu^{ij}(t)i}{dt} = \Omega_\nu^{il}\sigma_\nu^{lj}(t) + \Omega_\nu^{jm}\sigma_\nu^{mi}(t) + 2\mathcal{D}_\nu^{ij} \tag{7}$$

where σ_ν^{ij} is a symmetric matrix. The initial condition i.e at t= 0, $d\sigma_\nu/dt = 0$ ($\sigma_\nu = \sum_{i>j}\sigma_\nu^{ij}$) satisfies if $\mathcal{D}_\nu^{++} = \mathcal{D}_\nu^{--} = -\mathcal{D}_\nu^{+-}$. These equations are the same as those derived in Ref [8], the solutions of which provide a good fit to the simulation result [10].

Within the framework of Langevin or FP equation, further study can be directed to the non-linear regime of the dynamics by introducing non-linearity in the drift term. Since $-dU_\nu/dA_\nu$ correspond the drift coefficient of FP equation this can be done by adding a

quartic term to the associated quadratic potential. This stochastic model of non-linear Langevin equation, is rich in physics capable of explaining the general behaviour of the system in the presence of instability far from the equilibrium and onset of macroscopic structure [11]. The presence of the quartic term prevents an indefinite fall of a system and drive it towards any one of the two newly appeared stable fixed points which correspond to the minima of the dynamically generated cusps. The co-operative behaviour or the synergism of nonlinearity and random force simulate this typical feature i.e. *slowing down* the fluctuation near dynamical phase transition. The fluctuation deviate gradually from its initial Gaussian profile as the system approaches to the second or scaling regime. The modified equation for second moment with positive value of Ω is given by

$$\frac{d}{dt}\sigma_\nu^{++}(t) = 2\{\Omega_\nu - g_\nu\sigma_\nu^{++}(t)\}\sigma_\nu^{++} + 2\mathcal{D}_\nu^{++} \tag{8}$$

which provides a scaling solution

$$\sigma_\nu^{++}(t) = \frac{\Omega_\nu}{g_\nu}\frac{\tau}{1+\tau}\ ; \quad \tau = \frac{g_\nu}{\Omega_\nu}(\sigma_\nu^{++}(0) + \frac{\mathcal{D}_\nu^{++}}{\Omega_\nu})e^{2\Omega_\nu t} \tag{9}$$

with $g_\nu > 0$. However, in the derivation of Eq. (8) a physical approximation i.e. $A_\nu^{++3}(t) = \sigma_\nu^{++}(t)A_\nu^{++}(t)$ is made [11]. The time needed to reach the saturation for a unstable mode ν is given by

$$t_\nu^0 = \frac{1}{2\Omega_\nu} log[\frac{g_\nu}{\Omega_\nu}\{\sigma_\nu^{++}(0) + \frac{\mathcal{D}_\nu^{++}}{\Omega_\nu}\}]^{-1} \tag{10}$$

In Fig. 1. the open diamonds represents the time evolution of $\sigma_{\mathbf{k}}$ for unstable modes with values of k_x ranging from 0.3 to 0.6 with $k_y = 0$. For direct comparison to earlier results [3] we plot the function $L^2\sigma_{\mathbf{k}}$ (see Fig. 1.) so that the unit becomes fm^{-2}. The solution of Eqs. (7) (valid for linear regime of the dynamics) are shown by dotted curves. The solid curves represent the solution of non-linear generalization so that the evolution of ++ component is given by Eq. (9). For these solutions we use the initial condition i.e. at $t = 0$, $\langle A\rangle_\nu = \sigma_\nu = d\sigma_\nu/dt = 0$. The general agreement of our fit to the simulation result is quite good. The saturation time $t_{\mathbf{k}}^0$ for different modes are shown in the figure. The

most unstable mode i.e. the mode having minimum growth constant $t_{\mathbf{k}}$ ($= \Omega_{\mathbf{k}}^{-1}$) saturate earliest and the time $t_{\mathbf{k}}^{0}$ gradualy increses as the $t_{\mathbf{k}}$ increases. For some cases, the linear regime of the dynamics persists even at time $t = 120$ fm/c. Let us consider the evolution of density $\rho(x, y; t)$ in co-ordinate space. In earlier time up to $\approx$ 75fm/c when spatial average of $\sqrt{\langle(\delta\rho)^2\rangle}/\langle\rho\rangle$ reaches the value 1 we observe a considerable fluctuation in ρ in the individual samples. However, the sample average density i.e. $\langle\rho(x, y; t)\rangle$ is found to be almost uniform with $\sim 3\%$ fluctuation from its initial value of 0.5 which is essentially the mean (or single) trajectory result valid in the linear regime of the dynamics. As the time increases the non-linear effect makes the situation different. We see structure even in the sample average distribution of density which develops slowly in a coherent manner. In Fig. 2 we plot $\langle\rho(x, y; t)\rangle$ at a time $t = 150$fm/c. This observed structure almost remains steady with increasing time, which corresponds to a equilibrium or at least a metastable state. The fluctuation is seen to be $\sim 20\%$ with respect to the initial uniform state. The appearance of such spatial inhomogeneity in the average density is in conformation with that of a mean trajectory calculation reported in Ref. [12]. Though, the fluctuation in latter case is larger than what we observe here. This may be attributed to the different initial and also boundary conditions used in these two cases. However, large scale fluctuation exists in individual samples, as evident from the attainment of a non-zero steady value of $\sigma_{\mathbf{k}}$ in the present formalism.

In conclusion, using a recently proposed simulation scheme for inclusion of fluctuation in BUU dynamics we study the problem of spinodal decomposition in nuclear matter. Our work is mainly devoted to the study of the large time behaviour of system. To analyize the simulation result we apply the well known Suzuki's model of dynamical phase transition successfully in the context of spinodal decomposition. Accordingly, the evolution of the system is guided by the co-operative effect of non-linearity and fluctuation in a universal manner. To make our stand clear, we would like to mention a recent work done by Baldo et al. [13] where the evolution of fluctuation is studied in the framework of Vlasov equation. In

the simulation work they studied the growth of an initial disturbance imposed on the uniform nuclear matter. Without being saturated, the $\sigma_{\mathbf{k}}$ overshoots very fast from its linear trends (in log scale). It will be relevant here to note that the finite collision rate, no matter how small it be, changes the nature of the solutions describing a diffusive process as modeled by the Langevin equation. Therefore, the non-diffusive behaviour of fluctuation, although exhibits some non-standard evolution pattern, but does not indicate conclusively that the dynamics of fragmentation is dominantly non-linear. However, similar such studies facilitate us to extract the growth constant $t_{\mathbf{k}}$, (the RPA frequency) of the initial state [10] and also allow a further scope to check the consistency of our work. The values of $t_{\mathbf{k}}$ indicated in Fig. 1. provide a good fit to the dispersion relation shown in Fig 3. of Ref. [10].

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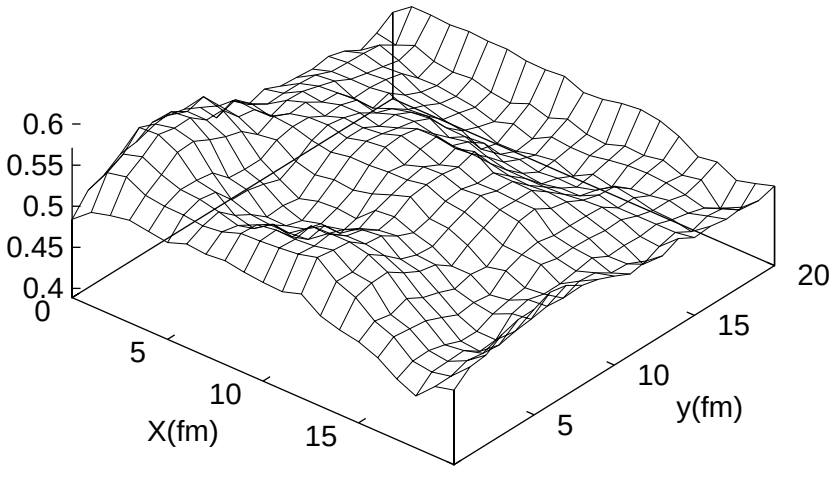
Figure Captions

Fig.1 The fluctuation coefficient $L^2\sigma_{\mathbf{k}}$ versus time t for different values of unstable mode $\mathbf{k}$ (k_x, k_y) are shown. The open diamonds correspond to simulation results in steps of 10 fm/c. The dashed curves represent the solution of Eq.(7) while solid curves show the nonlinear generalization of the same. The open circles represent the simulation result of a typical stable mode with ($k_x = 0.8$ fm^{-1}, $k_y = 0$) which exhibits an early saturation.

Fig.2 The sample average density profile $< \rho(x, y; t) >$ at time t= 150 fm/c versus the position (x, y) is shown.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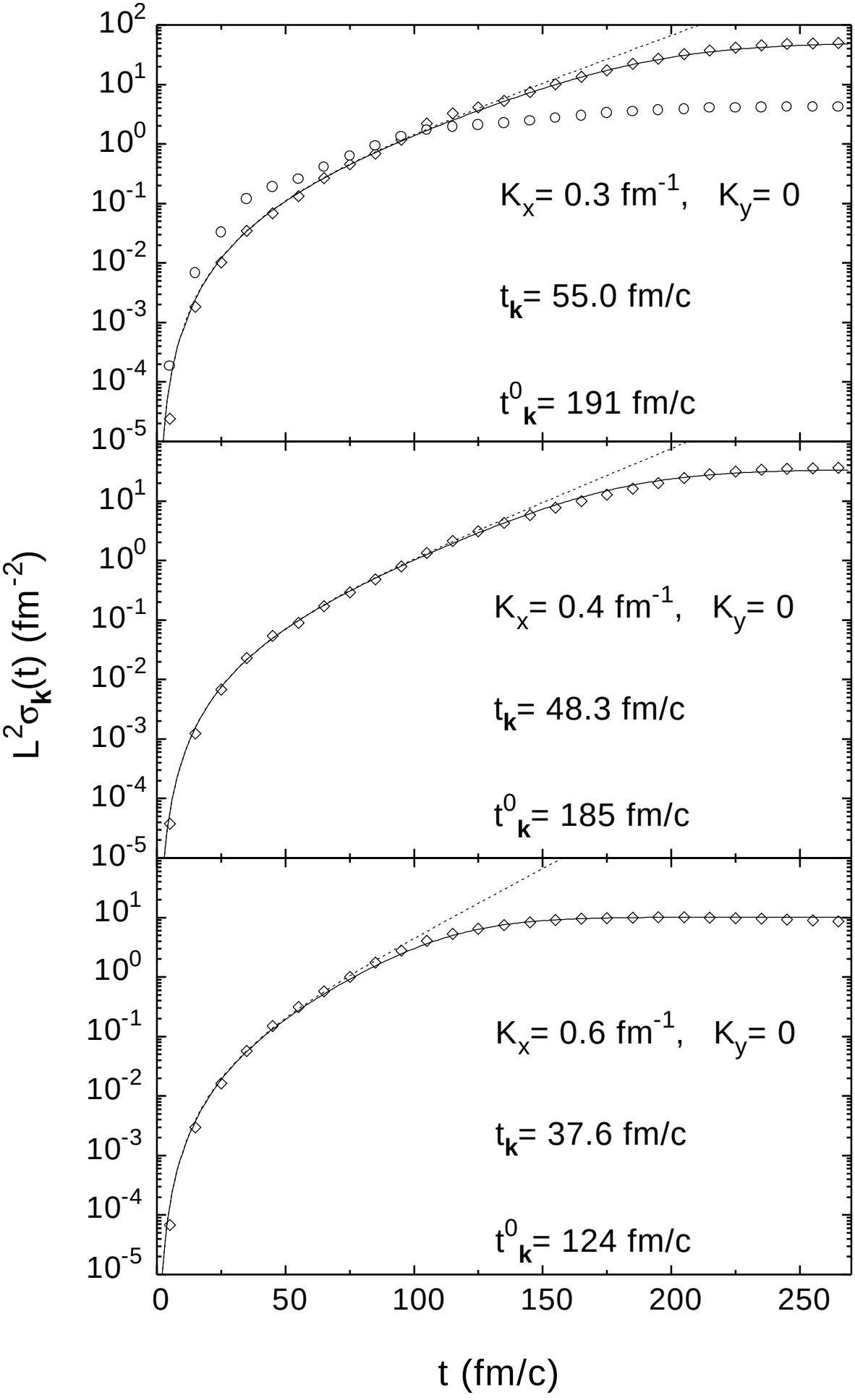
0.5
0.45
0.4
0
5
10
15
X(fm)
5
10
15
20
y(fm)


$L^2\sigma_k(t)$ (fm^{-2})
K_x= 0.3 fm^{-1}, K_y= 0
t_k= 55.0 fm/c
t^0_k= 191 fm/c
K_x= 0.4 fm^{-1}, K_y= 0
t_k= 48.3 fm/c
t^0_k= 185 fm/c
K_x= 0.6 fm^{-1}, K_y= 0
t_k= 37.6 fm/c
t^0_k= 124 fm/c
t (fm/c)